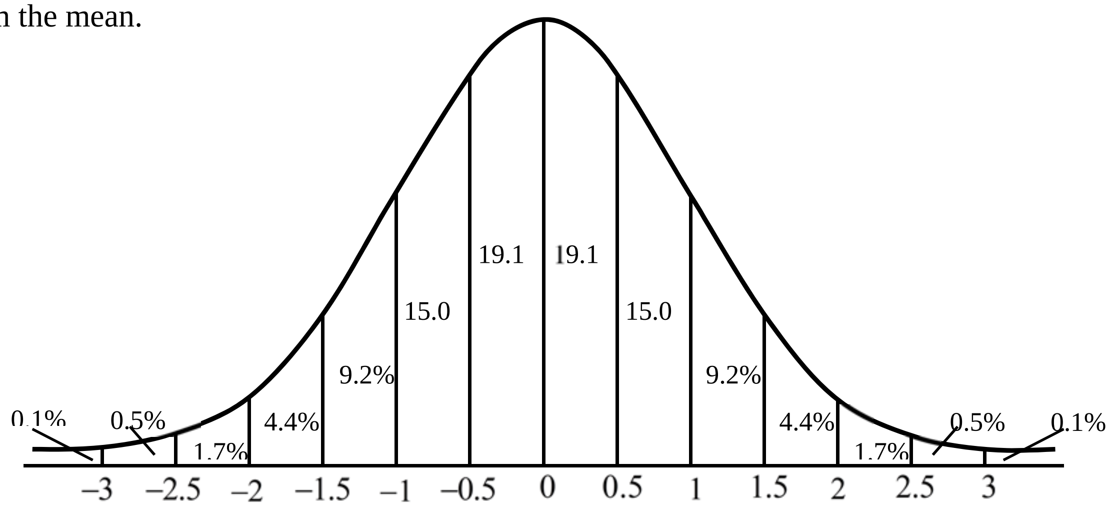
Name: ____________________________________ Date: _________________

A2CC: Normal Distributions and Z-scores

Many populations have a distribution that can be well described with what is known as **The Normal Distribution** or the **Bell Curve**. This curve, shows the percent or proportion of a normally distributed data set that lies certain amounts from the mean.



***Exercise* #1:** For a population that is normally distributed, find the percentage of the population that lies

(a) within one standard deviation of the mean.

(b) within two standard deviations of the mean.

(c) within three standard deviations of the mean.

The Empirical Rule:

As can be easily seen from *Exercise* #1, the majority of any normally distributed population will lie within one standard deviation of its mean and the vast majority will lie within two standard deviations. A whole variety of problems can be solved if we know that a population is normally distributed. The normal distribution can be used in increments other than half-standard deviations. In fact, we can use our calculators to determine probabilities (or proportions) for almost any data value within a normally distributed population, as long as we know the population mean, μ, and the population standard deviation, σ. But, first, we will introduce a concept known as a data value's z-score.

THE Z-SCORE OF A DATA VALUE

For a data point x_i, its z-score is calculated by: $z=\frac{x_i-\mu}{\sigma}$. It calculates how far from the mean, in terms of standard deviations, a data point lies. It can be positive if the data point lies above the mean or negative if the data point lies below the mean.

***Exercise* #2:** Boy's heights in seventh grade are normally distributed with a mean height of 62 inches and a standard deviation of 3.2 inches. Find z-scores, rounded to the nearest hundredth, for each of the following heights. Show the calculation that leads to your answer.

(a) $x_i = 66$ inches

(b) $x_i = 57$ inches

(c) $x_i = 70$ inches

However, our calculator can determine probabilities (or proportions) for almost any data value within a normally distributed population without us needing to calculate z scores at all!

We use a command called normalcdf which can be found under our distribution menu (2^{nd} Vars 2).

***Exercise* #3:** At Arlington High School, 424 juniors recently took the SAT exam. On the math portion of the exam, the mean score was 540 with a standard deviation of 80. If the scores on the exam were normally distributed, answer the following questions.

(a) What percent of the math scores fell between 500 and 660?

(b) How many scores fell between 500 and 660? Round your answer to the nearest whole number.

This process is sometimes used to determine a particular data point's **percentile**, which is the **percent of the population equal to or less than the data point**.

(c) If Evin scored a 740 on her math exam, what percent of the students who took the exam did better than her?

(d) Approximately how many students did better than Evin?

***Exercise* #4:** The heights of 16 year old teenage boys are normally distributed with a mean of 66 inches and a standard deviation of 3. If Jabari is 72 inches tall, which of the following is closest to his height's percentile rank?

(1) 85^{th}

(2) 67^{th}

(3) 98^{th}

(4) 93^{rd}

***Exercise* #5:** The amount of soda in a standard can is normally distributed with a mean of 12 ounces and a standard deviation of 0.6 ounces. If 250 soda cans were pulled by a company to check volume, how many would be expected to have less than 11.1 ounces in them?

(1) 17 (3) 28

(2) 23 (4) 11

***Exercise* #6:** Biologists are studying the weights of Red King Crabs in the Alaskan waters. They sample 16 crabs and compiled their weights, in pounds, as shown below.

9.8, 10.1, 11.1, 12.4, 11.8, 13.2, 12.8, 12.5, 13.7, 11.6, 13.4, 12.3, 12.6, 14.8, 14.2 15.1

(a) Determine the mean and sample standard deviation for this sample of crabs. Round both statistical measures to the nearest *tenth* of a pound.

(b) Why does this sample indicate that the population would be well modeled using a normal distribution? Explain. Hint – Use your calculator to sort this data in ascending order.

(c) Assuming your mean and standard deviation from part (a) apply to a normally distributed population of crabs caught in Alaska, what percent will fall between 9.6 pounds and 15.6 pounds?

(d) If fishermen must throw back any crab caught below 10.4 pounds, approximately what percent of the crabs caught will need to be thrown back if the weights are normally distributed?

***Exercise* #7:** If the scores on a standardized test are normally distributed with a mean of 560 and a standard deviation of 75. Answer the following questions by using z-scores and the normal distribution table.

(a) Find the probability that a test picked at random would have a score larger than 720. Round to the nearest hundredth of a percent.

(b) Find the probability that a completed test picked at random would have a score less than 500. Round to the nearest tenth of a percent.

(c) Find the probability that a completed test picked at random would have a score between 500 and 600.

(d) Find the probability that a completed test picked at random would have a score between 600 and 700.

There is another very helpful calculator command. It can find the data point that is at a specific percentile. It is called invNorm. It can be found under our distribution menu (2^{nd} Vars 3).

(e) What would a student have to score to have scored at the 95^{th} percentile?

(f) What score would be at the 15^{th} percentile?

Homework

Fluency

1. A population has a mean of $\mu = 24.8$ and a standard deviation of $\sigma = 4.2$. For each of the following data values, calculate the z-value to the nearest hundredth. You do *not* need to read the Normal table.

   (a) $x_i = 30$ (b) $x_i = 35$ (c) $x_i = 19$

   (d) $x_i = 15.4$ (e) $x_i = 24.8$ (f) $x_i = 33.2$

2. A population has a mean of 102.8 and a standard deviation of 15.4. If a data point has a z-value of 1.87 then which of the following is the value of the data point?

   (1) 28.8 (3) 131.6

   (2) 86.7 (4) 152.3

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Applications

Get practice with both the Normal Distribution Table and your calculator when doing the following problems.

3. A recent study found that the mean amount spent by individuals on a music service website was normally distributed with a mean of $384 with a standard deviation of $48. Which of the following gives the proportion of the individuals that spend more than $400?

   (1) 0.43 (3) 0.12

   (2) 0.74 (4) 0.37

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4. The hold time experienced by people calling a government agency was found to be normally distributed with a mean of 12.4 minutes and a standard deviation of 4.3 minutes. Which percent below represents the percent of calls answered in less than 5 minutes?

   (1) 4.3%   (3) 6.8%

   (2) 5.3%   (4) 12.9%

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5. The national average price per gallon for gasoline is normally distributed with a mean (currently) of $2.34 per gallon with a standard deviation of $0.26 per gallon. Which of the following represents the proportion of the gas prices that lie between $2.00 and $3.00?

   (1) 56%   (3) 84%

   (2) 72%   (4) 90%

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6. If the average teacher salary in the United States is $45,753 and salaries are normally distributed with a standard deviation of $7890, would a salary of $40,000 per year be in the lowest quintile of teacher salary? (Do a quick Internet search on the term quintile if you don't know what it means).

7. The average rent for a one bedroom apartment (in the Winter of 2015) in New York City is a whopping $2801 per month with a standard deviation of $920.

   (a) If rents are normally distributed, what percent of the apartments will be less than $2,500 per month?

   (b) If rents are normally distributed, how realistic is it to believe you will be able to rent a one-bedroom in New York City for less than $1,500 per month? Justify your answer.

   (c) A one-bedroom on the Upper East Side with a doorman and views of Central Park was listed at $5,000 per month. How rare is this? Assume the rents are normally distributed.

   (d) Do you think the rents are normally distributed? Keep in mind the normal distribution is symmetric about its mean (looks the same on both sides). If it isn't symmetric, what does it look like?

8. A national math competition advances to the second round only the top 5% of all participants based on scores from a first round exam. Their scores are normally distributed with a mean of 76.2 and a standard deviation of 17.1. What score, to the nearest whole number, would be necessary to make it to the second round? To start, look at the table and see if you can determine the z-value that corresponds to the top 5%.